PHASE TRANSITIONS AND FREE BOUNDARIES
FINAL REPORT

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OFFICE OF NAVAL RESEARCH

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INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
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FINAL REPORT

for

OFFICE OF NAVAL RESEARCH

1. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ONR SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   Mahmoud Affouf, Unique global solutions of initial-boundary value problems for thermodynamic phase transitions, IMA Preprint Series # 871.

   Mahmoud Affouf and J. Shatah, The computation of magnetic vortices in superconductors, UMSI preprint #91/209.

   Mahmoud Affouf, Numerical study of a singular system of conservation laws arising in enhanced oil reservoirs, IMA Preprint # 864.

2. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS PERIOD:

   Mahmoud Affouf

   WORKSHOPS SUPPORTED:

   1. Free Boundaries in Viscous Flows

   2. Degenerate Diffusions

   There will be proceedings volumes as a result of these two workshops. We are still collecting papers from the authors at this date. Completed volumes will be sent to the ONR.

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Papers received to date for the IMA Volume: Free Boundaries in Viscous Flows

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Papers received to date for the IMA Volume: Degenerate Diffusions

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IMA NEWSLETTER #175

March 9 - March 31, 1991

1990-91 Program
PHASE TRANSITIONS AND FREE BOUNDARIES

NEWS AND NOTES

IMA Workshop: FREE BOUNDARIES IN VISCOUS FLOWS

March 11–15, 1991
Organizers: R. Brown, S. Davis, S. Kistler

Viscous flows interact with liquid/fluid interfaces and solidification fronts in a wide variety of technologies including the processing of coatings, polymers, semiconductors single crystals and other advanced materials. This workshop addresses the mathematical treatment of the dynamical and instability phenomena in such flows. Special topics include solidification with flows, wetting lines, and the instabilities of drops and films. Presentations will focus on the development of asymptotic and numerical strategies for solution of these problems. Applications of analysis of complex flow problems will be high-lighted.

Most of the program talks will be held in Conference Hall 3-180 on the entry floor of the Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

LECTURES IN SUPERCONDUCTIVITY

Allen Goldman and John Ockendon will discuss macroscopic models of superconductivity and their relationship to free boundary problems in a lecture series at 1:25 pm, March 26 – 28, held in the Conference Hall EE/CS 3-180. Professor Goldman will distribute notes for his talks on the physics background of the models.

SCHEDULE FOR MARCH 9 - MARCH 31

Monday, March 11

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:00 am	**Registration and coffee**	Reception Room EE/CS 3-176
9:30 am	**Welcome and Orientation**	Conference Hall EE/CS 3-180
9:40 am	**Gerald W. Young** The University of Akron	Mathematical description of viscous free surface flows

Abstract: Viscous free surface flows are of importance to a wide variety of coating and material processing technologies. Designing, developing, and advancing these techniques requires quantitative understanding of the transport processes at hand, as well as the location, dynamics, and stability of the interface. Empirical investigations of these systems may be hindered by the presence of an unusually large number of independent processing parameters, or by difficulties in observing the complex and coupled interactions between the flows and the free surfaces. Further, there are situations where either the fundamental physics is poorly understood or only understood in isolation from one another. Hence, mathematical modeling is called upon to play an ever increasing role in addressing these issues. Forwarding the understanding of the science and assisting in multi-parameter optimization of the technology are key objectives.

This presentation primarily focuses on the formulation, and to a lesser extend the analysis and solution, of models simulating the interaction between viscous flows and free surfaces. Considerations in the modeling of liquid-fluid and liquid-solid interfacial system are discussed. A description of contact-line problems is also examined. Examples illustrating film flow, spreading, surface-tension-driven flow, and solidification phenomena are presented. Analysis of the models will concentrate on solution methodologies based upon asymptotic expansion techniques.

10:40 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Robert A. Brown** MIT	Mathematical description of viscous free surface flows

Abstract: Viscous free surface flows are described as the class of free- and moving- boundary problems in which a viscous flow must be computed in conjunction with the determination of the shape of either a liquid/fluid meniscus or a melt/solid phase boundary. The mathematical descriptions of these problems are stated with special emphasis on the conditions on the free boundaries and contact curves between these surfaces and other phases. Prototypical models of material processes with both melt/gas interfaces and solidification surfaces are used as examples.

The primary features of numerical algorithms for the solution of viscous free-surface flow problems are emphasized in the context of these models. The important issues emphasized are the spatial discretization of the flow equations, the effective representation of the interface and a numerical iteration scheme for simultaneous determination of the interface shape and the flow. These features are addressed fully in the talks by others at this meeting.

Numerical Analysis Seminar

1:25 pm	**Bernardo Cockburn** University of Minnesota	Discontinuous Galerkin methods for 2D gas dynamics

The SEMINAR meets in Vincent Hall 570

2:00 pm	**Josef Bemelmans** Saarbrücken	Free-boundary value problems for the stationary Navier-Stokes equations

Abstract: The motion of an incompressible viscous fluid that occupies a given domain Ω whose boundary Σ is governed by surface tension or self-attraction is described by the following free-boundary problem for the Navier-Stokes equations:

$$(N) \qquad \begin{array}{r} -v\Delta v + Dp + (v \cdot D)v = \rho \\ \operatorname{div} v = 0 \end{array} \text{ in } \Omega$$

$$(m) \qquad v \cdot n = 0,\ t_k \cdot T(v,p) \cdot n = 0 \text{ on } \Sigma,\ k = 1,2$$

$$(H) \qquad 2KH = n \cdot T(v,p) \cdot n \text{ on } \Sigma$$

or

$$(G) \qquad \int_\Omega \frac{g}{|x-y|} dy = n \cdot T(v,p) \cdot n \text{ on } \Sigma.$$

Condition (H) states that the normal component of the stress vector is proportional to the m ean curvature H of Σ; if the fluid body is held together by self-attraction instead of surface tension, then condition (G) is imposed.

We discuss existence, uniqueness, and regularity of solutions to the boundary value problems (N), (m), (H) and (N), (m), (G). For the latter one hard implicit function theorems are a basic tool. The solutions to these problems can be considered to be generalizations of classical equilibrium figures of rotating liquids. This connection leads to interesting open questions.

3:00 pm	**Elizabeth N. Dussan** Schlumberger Doll Research Center	Wetting lines
4:00 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

Tuesday, March 12

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Marc K. Smith** The Johns Hopkins University	Longwave interfacial instabilities in viscous liquid films

Abstract: A thin film of viscous liquid is susceptible to many different kinds of instabilities. One well-known mode of instability displays a wave-like deformation of the interface of the liquid film with a wavelength that is much larger than the thickness of the film. We shall examine this instability and show how the dynamics of the instability mechanism can be explained by carefully interpreting the mathematical solution of the instability problem obtained by using a standard regular perturbation technique.

Several different geometries will be considered. The most fundamental is the isothermal thin film on an inclined plane. Here, we explain how the small effects of liquid inertia lead to the unstable interfacial motion. Next, we consider both heating and cooling the layer from below and show the effects of buoyancy forces and direct liquid expansion. For an annular film in a vertical pipe, we show how the unstable behavior of the film is modified by a large lubrication pressure that appears because of the rigid pipe wall. Finally, a horizontal layer driven by thermocapillary is considered to demonstrate the effect surface-tension gradients have on the instability.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**David S. Dandy** Sandia Nat. Labs.	Finite-difference and finite-volume techniques for intermediate Reynolds number flows

Abstract: Two approximation techniques for solving finite Reynolds number (low Mach number) free-surface flows in two and three dimensions will be compared and contrasted. The first, finite-difference, will be applied

to the buoyancy-driven motion of an axisymmetric deformable bubble. In this treatment, the interface corresponds to a coordinate surface in a boundary-fitted system, and its position is treated as unknown. The governing set of partial differential equations for the coordinate mapping and the flow are approximated by second-order accurate, centered finite-differences, resulting in a set of coupled nonlinear algebraic equations which is then solved using Newton's method. Several sparse matrix algorithms have been applied to the linear Newton system, and these will be described.

The second approximation technique, finite-volume, will be employed on three, three-dimensional nonisothermal flows: a sphere in linear shear, a prolate spheroid in uniform flow at angle of attack, and flow in a curved tube. Three-dimensional, body-fitted curvilinear coordinates yield cell-centered volume elements having four, five, or six sides. The conservation equations, written in integral form, are approximated within each volume element. The volume and surface integral approximations give rise to a set of coupled nonlinear algebraic equations. Due to computer memory limitations, the system of equations is currently solved using a predictor-corrector algorithm that has been optimized for fast convergence, but work is underway to apply Newton's method to the system of equations.

2:00 pm	**Lee-Wing Ho** Nektonics, Inc., Cambridge MA	Imposition of surface-tension boundary conditions in finite simulation of three-dimensional viscous free-surface flows

Abstract: Over the last two decades the finite element method has been applied successfully to the simulation of many free-surface fluid flow problems of fundamental and practical importance. The success of the method can be attributed to the use of variational formulation and domain decomposition techniques. While the former allows simple treatment of variable properties as well as natural imposition of Neumann type boundary conditions, the latter enables the modelling of complex and time-dependent flow geometry in both two and three space dimensions.

In this study, a new variational form is proposed for the imposition of general surface-tension boundary conditions for three-dimensional incompressible viscous free-surface flows. This new form has the following important advantages: *first, it provides a consistent treatment for variable surface tension*; second, it automatically generates (in a weak sense) natural conditions for C^1-continuity of the free-surface geometry; and lastly, it is entirely surface-intrinsic and thus well-suited for finite element discretization. In the present study this new variational form is discretized spatially using spectral element (high-order or p-type finite element) method and temporally using semi-implicit time-stepping schemes; the resulting system of algebraic equations are inverted using preconditioned conjugate gradient iteration. Several examples are given as a demonstration of the effectiveness of the approach.

3:00 pm	**Coffee Break**	Reception Room EE/CS 3-176
3:30 pm		**Contributed Papers**

Participants are invited to present contributed papers. Twenty minute time slots have been set aside for these talks. It is hoped that the speakers will leave five minutes of the time for discussion. Please contact the organizers if you wish to give a contributed paper.

3:30 pm	**Ulrich Hornung** U. der Bundeswehr München	**Some examples of capillary surfaces**
3:50 pm	**Vladislav Pukhnachov** USSR Acad. Sci., Novosibirsk	**Anomalous Marangoni effect in its manifestation in thin liquid layers**
4:10 pm	**Dan Joseph** University of Minnesota	**Expansion velocity and Korteweg stresses in simple mixtures of incompressible fluids**

Wednesday, March 13

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Stephan F. Kistler** 3M	Free boundaries in viscous flows

Abstract: In the computational analysis of viscous free-surface flows, coordinate representation and coupling between flow and free-surface calculations are major challenges. Substantial changes in macroscopic flow geometry can be accompanied by the formation of highly curved interfaces in the immediate vicinity of apparent singularities, such as those that arise near cusp-like corners and dynamic wetting lines. After a brief review of previously published Galerkin finite-element techniques for solving viscous free-surface flows, this presentation focuses on a generalized algebraic mesh generation scheme that can track major changes in the macroscopic flow domain, yet simultaneously permits extensive mesh refinement near singularities. Several case studies of coating flow with dynamic wetting lines illustrate the capabilities of the algorithm. Computed predictions are shown to agree with experimental measurements and, where available, with results from matched asymptotic expansions.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Jonathan J. L. Higdon** University of Illinois, Urbana	High order boundary integral methods for Stokes flow

Abstract: We consider the development of high order implementations of the boundary integral method for Stokes flow. Practical issues of discretization, quadrature and matrix inversion are discussed and a general approach for three dimensional flows is presented. For free surface flows, the general algorithm may be supplemented with time stepping algorithms for transient problems or a Newton Raphson scheme for equilibrium configurations.

2:00 pm	**Ole Hassager** DTH Lyngby DENMARK	Methods for solving free surface problems

Abstract: The talk will focus on the application of the finite element method for the solution of viscous free surface problems. It will be illustrated how models for interfacial tension and contact angles may be implemented in a natural way in the variational basis for the finite element method. Time dependent flows and moving meshes will be described. Results will be given for specific flow problems such as the motion of drops in capillaries, and coating flows. Problems associated with moving contact lines will be discussed.

3:00 pm	**Ridgway Scott** University of Houston	Asymptotic and numerical methods for approximating flows with a free boundary

Abstract: The phenomenon which motivates the current work is the flow of a liquid constrained only partly by a container, that is, in which a part of the boundary of the domain filled by the liquid is an interface with another liquid of much smaller density. One model for the behavior of such liquids is based on the assumption that the surface tension between the two liquids is proportional to the curvature of the free surface. This model has been studied extensively in recently years, both experimentally, theoretically, and computationally. We will describe both asymptotic and numerical methods for approximating the solution. The asymptotic methods are based on the so-called lubrication approximation, but one in which inertial terms are included. The numerical techniques are based on finite element approximations of the Navier-Stokes equations in two dimensions. Both the asymptotic and numerical methods show remarkable accuracy. A further objective of the talk is to describe a framework for the analysis of convergence properties of the computational techniques being used. We will describe for a model problem how the numerical approximation of the free boundary affects the approximation of the other variables of the problem and *vice-versa*.

Thursday, March 14

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**S. R. Coriell** NIST	Interaction between fluid flow and morphological stability

Abstract: During freezing of a liquid phase the crystal-melt interface is subject to morphological instability leading to cellular and dendritic growth. Fluid flow in the melt can alter the conditions for the onset of morphological instability. Classic hydrodynamic instabilities can be altered by the presence of a crystal-melt interface (free boundary). The interaction between fluid flow and the crystal-melt interface is an active area of research. Examples of these interactions based primarily on linear stability analyses of the coupled system will be discussed. For example, if a lighter solute is rejected during directional solidification vertically upwards, both convective and morphological instabilities can occur.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176

11:00 am	**M. Grae Worster** Northwestern University	Interfaces with mushy layers

Abstract: Solidification fronts in binary alloys are prone to morphological instabilities that prevent the front from being planar in many situations. Stability analyses can determine the onset of morphological instability and small, finite-amplitude analyses can follow the evolution of the interface under weakly supercritical conditions. Expensive and time-consuming numerical simulations can predict the formation of deep cells and some of the cell-splitting phenomena that begin to deform the interface on smaller and smaller lengthscales. However, once the solid-liquid interface has become highly convoluted, perhaps dendritic, it is no longer practical to attempt the prediction of its precise morphology. Instead, the study of mushy layers seeks to provide an averaged description of the resultant two-phase medium (solid dendrites and interstitial liquid) on lengthscales larger than the fine-scale structure of the solid-liquid interface. The problem is thus converted from a two-phase problem (solid and liquid) with a single, complicated interface into a three-phase problem (solid, mush and liquid) with two geometrically simpler interfaces. However, this simplification carries the price of having to determine appropriate boundary conditions for the interfaces of an averaged medium. The seminar will comprise a general discussion of the theory of mushy layers with particular attention paid to the thermodynamic interfacial conditions and the ways in which these modify or are modified by natural convection in the melt and through the interstices of the mushy layer.

2:00 pm	**Jeffrey J. Derby** University of Minnesota	Macroscopic interactions of viscous flow, heat transfer, and solidification interfaces in melt crystal growth systems

Abstract: We are entering an era in which chemical engineers and materials scientists are attempting to influence the properties of advanced materials by control at the molecular level. In solidification systems, this control requires detailed understanding of the fundamentals of morphological stability and the effects of macroscopic transport in the processing systems which are used to produce these materials. This seminar will address some issues in understanding the growth of large-dimension laser host crystals. We will detail the development of and present recent results from mathematical models of the Bridgman and Czochralski crystal growth processes.

3:00 pm	**Michael M. Chen** University of Illinois, Urbana	Generic structure of flow and temperature fields in welding and high energy beam processing

Abstract: Thermocapillary convection plays a dominant role in the fluid flow and heat transfer of a number of materials processing techniques, especially welding and other processing modalities involving melting due to concentrated energy beams, such as laser and electron beam processing and plasma processing. Typically, these processes produce a pool of molten material with intense temperature gradients along the surface, and it is the surface tension gradient associated with the surface temperature variation which drives the flow. The dimensionless parameter for thermocapillary convection is the Marangoni number, which can be viewed as the Peclet number based on the elementary thermocapillary velocity scale. For most of the processes

cited above, the Marangoni number is of the order of 10^4 or more. Thus the flow is expected to possess singular regions, including thermal and viscous boundary layers and a unique thermocapillary singularity at the corners. These singular regions render numerical computations extremely challenging. In addition, common numerical approximations, such as upwind differencing, can be particularly damaging because of the phenomenon's strong dependence on accurate temperature gradients in these singular regions. Failure to appreciate these difficulties often leads to inaccurate or erroneous numerical results and hence incorrect conclusions concerning processing. In this paper the physics of thermocapillary convection under common materials processing conditions will be described. The structure of temperature and velocity distributions, the scalings of the boundary layers and corner singular regions, and their known implications on material processing will be discussed. In addition, common pitfalls for numerical computations of such flows will be reviewed.

4:00 pm	**Akerman Hall 130D**	Tour of Dan Joseph's laboratory

We will show experiments on water lubricated pipelining with bamboo waves and corkscrew waves, two dimensional cusped interfaces which violate Laplace's law, rollers, drafting, kissing and tumbling of fluidized particles, non linear stabilization of fingering instabilities in porous media. The tour will probably last 30 or 40 minutes. **Please sign up for this tour in the Reception Room, EE/CS 3-176.**

5:45 pm	**Workshop Reception** & Buffet Dinner	Campus Club, 4th floor, The Terrace, Coffman Union

Reservations required. The reception begins at 5:45 (wine and cheese) followed by dinner at 6:30.

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Friday, March 15

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Saleh Tanveer** Ohio State University	Asymptotic techniques in Hele-Shaw flows

Abstract: The motion of a less viscous fluid displacing a more viscous fluid in a Hele-Shaw cell under the action of a pressure gradient has been the subject of intensive research over the last few years because of the mathematical analogies to two phase flow in a porous medium, dendritic crystal growth, directional solidification and electrochemical growth. In this talk, we will describe some of the analytical techniques that have proved useful in answering questions on steady state solutions, their linear stability and more recently in the complete time evolution problem for small surface tension at the level of formal asymptotics. The key role of asymptotics beyond all orders will be delineated.

10:30 am	**Coffee Break**	Vincent Hall 502

11:00 am	**L. E. Scriven** University of Minnesota	Theory of viscous free surface flows: Computing steady states, stability and sensitivity

Abstract: Viscous flows of the sort in precision coating operations are made nonlinear by their free surfaces. Inertial and non-Newtonian behavior can contribute as well. Successfully managed, such flows though contorted are steady and two-dimensional apart from edge effects. But they are prey to barring, ribbing and other unwanted instabilities. These are influenced not only by the flow parameters but also by the coater configuration. Sensitivities to persistent external disturbances are likewise important in practice.

Accordingly we have in a series of researches, most notably those carried out by K.N. Christodoulou, solved Navier-Stokes-type systems by Galerkin's method of weighted residuals and representation in terms of finite element basis functions on subdomains, the latter generated with algebraic or elliptic mesh design equations and related to a cartesian computational domain by isoparametric or subparametric mapping that accommodates free surfaces.

Steady states we find from the resulting nonlinear system $A(x)x = b$ by Newton iteration or variants, with initialization by first-order continuation and optimization of parameter step size. Responses to small

transitory distrubances we find from linear stability theory, which leads to a large, sparse, asymmetric, generalized eigenproblem

$$Jx = \lambda M x$$

in which J and M depend on system parameters and M is singular. We solve for leading modes – eigenvalues of algebraically largest real parts, and their eigenvectors – by a flexible method assembled from the iterative Arnoldi algorithm with Schur-Wielandt deflation, initialization that incorporates rational acceleration to eliminate infinite eigenvalues resulting from algebraic constraints, complex or real shift of eigenvalue as appropriate, and approximately exponential preconditioning by rational transformation suitable to the singular behavior.

Responses of stable states to persistent periodic disturbances we find from linear frequency response theory, which leads to a doubly large, sparse linear system

$$\begin{bmatrix} j & -\omega M \\ \omega M & j \end{bmatrix} \begin{bmatrix} Re & (z) \\ IM & (z) \end{bmatrix} = \begin{bmatrix} -F \\ 0 \end{bmatrix}$$

in which ω is the forcing frequency and F the sensitivity vector with respect to the forcing p meter. This we solve efficiently for amplification versus frequency. When nonlinear transient response is needed we employ the Gresho-Lee-Sani algorithm or in some circumstances the DASSL algorithm.

For parsimonious probing of parameter space we trace steady states marked by turning points (fold tracking) and other critical features such as sensitivity thresholds, ordinary and Hopf bifurcations. To do this we solve by Newton-like methods and continuation the steady-state equations augmented with a set that uniquely characterizes the critical feature. No less complete a theory is needed to optimize configuration and operation of many other processes as well.

References

S.F. Kistler & L. E. Scriven, Coating flow theory by finite element and asymptotic analysis of the Navier-Stokes system, Int. J. Num. Meth. Fluids 4 207 (1984)

K.N. Christodoulou & L.E. Scriven, Finding leading modes of a viscous free surface flow: an asymmetric generalized eigenproblem, J. Sci. Comp. 3 355-406 (1988)

S.F. Kistler & L. E. Scriven, The teapot effect: sheet-forming flows with deflection, wetting and hysteresis, J.Fl. Mech., in press (1991); K. N. Christodoulou & L.E. Scriven Operability limits of free surface flow systems by solving by solving nonlinear eigenvalue problems, Int. J. Num. Meth. Fluids (to appear).

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

2:00 pm	**José Francisco Rodrigues** Univ. Lisbon & CMAF	The two-phase steady-state Stefan problem in strips

Abstract: In this joint work with B. Faltzman, we show the steady-state free boundary of a Stefan problem of continuous casting type, in two dimensions, is given by a Lipschitz wave. This implies the existence of a classical solution in an almost everywhere sense. We establish the stability of this wave with respect to the length L of the domain and by letting $L \to +\infty$, we cover the case of an infinite strip. We obtain an error of order $1/\sqrt{2}$ for the difference, in the C^1-norm, of the temperatures and of the free boundaries associated with the finite and infinite strips, respectively.

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Monday, March 18

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**J. Thomas Beale** Duke University	Exact solitary water waves with capillary ripples at infinity

Abstract: We discuss solitary waves as exact solutions of the fluid equations with a free surface above. If surface tension is neglected, it has long been known that there are exact traveling waves whose first approximation is the famous soliton of the Korteweg-de Vries equation. However, if surface tension is included, a resonance occurs with periodic waves of the same speed. The corresponding waveform consists of a single crest on the elongated KdV scale with a much smaller oscillation at infinity on the physical scale. We have not proved that the amplitude of the oscillation is actually nonzero, but a formal calculation suggests that it is exponentially small. We will describe the two types of wave motion involved and then discuss the techniques used in proving the existence of exact solutions. A similar result has been obtained by S.M. Sun and M.C. Shen.

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Tuesday, March 19

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

11:15 am **Vladislav Pukhnachov** — Multi-dimensional self-similar solutions of parabolic equations
Lavrentyev Institute of Hydrodynamics

Abstract: We consider the parabolic equation

1 $$u_t = \Delta(u^m)$$

where Δ is the Laplacian acting on the spatial variables $(x_1, \dots, x_n) = x$ and m is a positive number. It is well known that equation (1) has a two-dimensional group of scaling transformations. This property allows to search for self-similar solutions of (1).

The first type of solution has the form

2 $$u = t^\beta f(x_1/t^\alpha, \dots, x_n/t^\alpha),$$

where $\beta = (2\alpha - 1)(m-1)^{-1}$ and α is an arbitrary real number if $m \neq 1$. In the linear case ($m = 1$) we may consider β to be an arbitrary real number in (2) but α must be equal to 1/2.

Self-similar solutions of the second type exist for the non-linear equation (1) only, i.e., when $m \neq 1$. These solutions have a representation

3 $$u = r^\gamma g(\theta_1, \dots, \theta_{n-1}, t).$$

Here $\gamma = 2(m-1)^{-1}$, $r = |x|$, and $\theta_1, \dots, \theta_{n-1}$ are spherical coordinates in space x. The deep difference between the solution types follows from the fact that the function f in (2) satisfies an elliptic equation on the variables $\zeta_i = x_i/t^\alpha (i = 1, \dots, n)$; on the contrary, the equation for function g in (3) remains parabolic.

In this report we present the approach for study of both solution types of equation (1). It is based on the transformation of (1) to new independent variables (Lagrangian coordinates). The Lagrangian coordinates method for investigation of multi-dimensional parabolic equations was proposed by A.M. Meirmanov and V.V. Pukhnachov in 1980 and independently by M.E. Gurtin, R.C. McCamey and E.A. Socolovsky in 1984. This method, in particular, allows the transformation of the free-boundary problem for (1) in the case $m > 1$ and non-negative initial data $u_0(x)$ (vanishing possibly on some region) into the problem in a fixed domain for some quasilinear system of differential equations of mixed type.

Using the above mentioned method, S.I. Shmarev proved in 1986 existence of self-similar solutions for the one-phase Stefan problem for the linear heat equation (namely, equation (1) when $m = 1$). This solution describes the situation where liquid phase is initially contained in the cone in R^n and the initial temperature distribution is a uniform function on $x_1, \dots, x_n$ of zero homogeniety degree. This example leads to a solution of the first type. We study here the asymptotic behavior of the Shmarev solution for small and large values of the self-similar variable $|x|/t^{1/2}$ and show in particular that the free boundary has a conical point in the origin for all $t > 0$.

For the non-linear equation (1) with $m > 1$ we give a simple proof of the uniqueness of the solution of first-type with compact support in the space of the variables $\zeta_i = x_i/t^\alpha (i = 1, \ldots, n)$. Such self-similar solutions are possible only when $\alpha = \alpha_* \equiv [n(m-1)+2]^{-1}$ and they are exhausted by the well-known Barenblatt solutions.

A more complicated situation arises for first-type solutions corresponding to initial data with noncompact (in fact, conical) support. We demand here some regularity on the self-similar solution at the origin but allow it *to have power growth when* $|\zeta| \equiv |x|/t^\alpha \to \infty$. *It turns out that these solutions do not exist if* $\alpha \leq \alpha_*$. The question of their existence for $\alpha > \alpha_*$ remains open. However, the solvability of the corresponding linearized problem and the investigation of asymptotic properties of the solution as $x\zeta \to 0$ and $\zeta \to \infty$ allows hope for a positive answer to this question.

As an example of the second type of self-similar solutions we study the Cauchy problem for the Boussinesq equation in the filtration theory. This case corresponds to $n = 2$ and $m = 2$ in equation (1). Here the initial function is $u_0 = r^2 g_0(\theta)$ (r and θ are polar coordinates in the plane). The non-negative function g_0 equals zero outside the interval $[-\delta, \delta]$, where $\delta \in (0, \pi)$. In this case the free boundary consists of two rays, $\theta = \pm\lambda(t)$, where $\lambda(0) = \delta$. Our guess is that as t increases the function $\lambda(t)$ increases up to the value π and at a later time the solution blows up. This hypothesis is confirmed by numerical simulation of the problem.

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Wednesday, March 20

SEMINAR IN { Open Problems in Applied Mathematics / Vincent Hall 570 }

4:45-5:30 pm **Hans van Duijn**
Delft University of Technology

Background: *This is a series of discussions on open problems in applied and industrial mathematics.* IMA visitors, especially post-docs, are encouraged to come.

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Thursday, March 21

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Wei-Ming Ni** University of Minnesota	Spike layers in singular perturbation theory

Abstract: I shall try to give a brief survey of recent progress on spike layer solutions in singular perturbation problems

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Monday, March 25

Numerical Analysis Seminar

1:25 pm	**Alfio Quarteroni** University of Minnesota	Domain decomposition for partial differential equations

The SEMINAR meets in Vincent Hall 570

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

2:30 pm	**Viorel Barbu** University of Cincinnati	Controlling the free boundary of the obstacle problem in a variable domain

Abstract: One proves that the free boundary of the variational inequality

$$\begin{aligned}
&\Delta y = f \text{ in } \{y > 0\} \\
&\Delta y \leq f \text{ in } \Omega_u = \{(x,y) \in R^2; 0 < x < 1, u(x) < y < A\} \\
&\quad y \geq 0 \text{ in } \Omega_u \\
&\quad y = 0 \text{ in } \{y = 1\}, y = 1 \text{ in } \{y = u(x)\}, y_x = 0 \text{ in } x = 0, 1
\end{aligned}$$

in the variable domain Ω_u, $u \in H^3(0,1), u < A$ is approximately controllable.

A similar result remains true for the corresponding parabolic problem.

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Tuesday, March 26

SEMINAR IN { Free Boundary Problems, Vincent Hall 570 }

11:15 am	**Mitchell Luskin** University of Minnesota	Numerical modeling of the microstructure of crystals with symmetry-related variants

Abstract: A continuum theory for crystals which predicts the existence of stable, symmetry-related martensitic variants, phase boundaries, and fine-scale twinning has mathematical features very unlike those usually considered in continuum mechanics. In particular, solutions to the appropriate variational problems must be described in terms of a material microstructure rather than in terms of a deformation. The numerical computation of approximate solutions to the continuum models is difficult because the bulk energy is nonconvex and has many local minima and because the solution is highly oscillatory. For these reasons, we have developed new concepts to interpret the numerical approximation of this microstructure .

We will give computational results and numerical analysis for continuum models for crystals with symmetry-related (martensitic) variants. These models relate the experimentally observed microstructure in shape memory alloys such as CuZn, CuAlNi, NiAl, and InTl to the numerical minimization of bulk energy.

LECTURES IN { Superconductivity, Conference Hall EE/CS 3-180 }

1:25 pm	**Allen Goldman** University of Minnesota	Macroscopic theories of superconductivity: London and Ginzburg-Landau models, I

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Wednesday, March 27

LECTURES IN { Superconductivity, Conference Hall EE/CS 3-180 }

1:25 pm	**Allen Goldman** University of Minnesota	Macroscopic theories of superconductivity: London and Ginzburg-Landau models, II
2:45 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

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Thursday, March 28

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

9:00 am	**Walter Littman** University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates is meeting on a regular basis with Professor W. Littman in Vincent Hall 570 in conjunction with the Industrial Problems Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific areas.

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Hans van Duijn** Delft Univ. of Technology/IMA	A free boundary problem involving a cusp

Abstract: We consider the behaviour of the interface between fresh and salt water in a porous medium (reservoir). The salt water is below the interface (w.r.t. the direction of gravity) and is stagnant. The fresh water is above the interface and moves towards the wells which are present in the reservoir. We give a description of the corresponding flow problem leading to a weak variational formulation involving a parameter Q which is related to the strength of the wells. We show that Q is a critical parameter in the following sense: there exists $Q' > 0$ such that for $Q < Q'$ a smooth interface exists which is monotone w.r.t. Q. For $Q = Q'$ a free boundary with one or more singularities (cusps) will occur at a positive distance from the wells. The global analysis for the problem (uniqueness, monotonicity and existence) is three dimensional. The local cusp analysis is two dimensional.

This is joint work with H.W. Alt from Bonn University.

LECTURES IN { Superconductivity / Conference Hall EE/CS 3-180 }

1:25 pm	**J. R. Ockendon** University of Oxford	Macroscopic superconductivity modelling

CURRENT IMA PARTICIPANTS

POSTDOCTORAL MEMBERS FOR 1990-91 PROGRAM YEAR

NAME	PREVIOUS/PRESENT INSTITUTION
Affouf, Mahmoud	New Jersey Institute of Technology
Altschuler, Steve	University of California, San Diego
Chen, Chao-Nien	Indiana University
Choe, Hi Jun	University of Kentucky
Fan, Haitao	Brown University
Fehribach, Joseph	University of Alabama, Huntsville
Filippas, Stathis	Courant Institute
Firoozye, Nikan	Courant Institute
Mou, Libin H.	Rice University
Smereka, Peter	Courant Institute

POSTDOCTORAL MEMBERSHIPS IN INDUSTRIAL MATHEMATICS FOR 1990-91 YEAR

NAME	PREVIOUS/PRESENT INSTITUTION
Dobson, David C.	Rice University
Fatemi, Emad	University of California, Los Angeles
Hoffend, Thomas R. Jr.	State University of New York, Buffalo
Xu, Yongzhi	University of Delaware

LONG-TERM VISITORS IN RESIDENCE
4 Weeks or Longer

Aguilera, Nestor E.	Inst. de Desarrollo Tec	Mar 1 - April 30
Aronson, Donald	University of Minnesota	Sept 1 - June 30
Bauman, Patricia	Purdue University	Sept 1 - June 30
Bemelmans, Josef	U. Saarbrucken	Jan 21 - Jan 25 Feb 25 - Mar 21
Bernis, Francisco	U. Complutense, Madrid	Feb 20 - June 30
Brule, Tim	University of Minnesota	
Davis, Steven	University of Minnesota	
Egnell, Henrik	University of Minnesota	
Escobedo, Miguel	Northwestern University	Mar 25 - April 21
Fabes, Gene	University of Minnesota	Sept 1 - June 30
Fosdick, Roger	University of Minnesota	Sept 1 - June 30
Friedman, Avner	IMA	
Guetter, Art	Hamline University	Jan 15 - June 15
Gulliver, Robert	University of Minnesota	Sept 1 - June 30
Guo, Jong-Sheng	National Tsing Hua U.	Mar 29 - April 30
Hoffman, Karl-Heinz	U. Augsburg	March 1 - April 30(one month)
Huang, Shaoyun	Peking University	March 15 - June 15
James, Richard	University of Minnesota	
Jetto, Becky	University of Minnesota	
Jimbo, Shuichi	Okayama University	Jan 28 - March 20
Jong-Pau, Yang Xiong	University of Minnesota	
Jossart, Paula	University of Minnesota	
Kamin, Shoshana	Tel-Aviv University	Feb 24 - June 30
Li, Dening	West Virginia University	Jan 1 - April 30
Li, Tong	Princeton University	Feb 10 - June 30
Liljeberg, John	University of Minnesota	
Littman, Walter	University of Minnesota	Sept 1 - June 30
Loe, Brian	Carlton College	Sept 1 - June 30
Lunel, Sjoerd Verduyn	Georgia Institute of Technology	April 1-30, June 1-30
Luskin, Mitchell	University of Minnesota	
McFadden, Geoffrey	NIST	Sept 1-Jan 4, Mar 11-15
McLeod, Bryce	University of Pittsburgh/Oxford	Mar 1 - April 30
Meirmanov, A.M.	Novosibirsk State University	Mar 6- May 6
Meyers, Norman	University of Minnesota	Sept 1 - June 30
Miller, Willard, Jr.	IMA	
Mukherjee, S.	Mahatma Gandhi Degree Coll.	Aug 27 (90) - Aug 26, (91)
Ni, Wei-Ming	University of Minnesota	
Nitsche, Johannes	University of Minnesota	
Olver, Peter	University of Minnesota	
Peletier, L. A.	University of Leiden	Feb 1 - June 9
Phillips, Daniel	Purdue University	Sept 1 - June 30
Promislow, Keith	Indiana University	Mar 1 - Mar 31
Pukhnachov, V.V.	USSR Acad. of Sciences	Jan 31 - Mar 31
Pukhnachov, Tatyana	USSR Acad. of Sciences	Jan 31 - Mar 31
Sakaguchi, Shigeru	Tokyo Inst. of Technology	Aug 27 - June 30
Serrin, James	University of Minnesota	Jan 1 - June 30
Solonnikov, V.A.	LOMI, USSR	March 9 - April 8
Spruck, Joel	U. of Massachusetts	Mar 1 - April 30
Suzuki, Takashi	Tokyo Metro University	Mar 27 - April 28
van Duijn, Hans	Deft Univ. of Technology	March 1 - June 30

Vazquez, Juan Luis	U. Autonoma, Madrid	Feb 17 - Aug 31
Vernescu, Bogdan	Inst. of Math-Bucharest	Aug 31 - June 30
Wang, Lihe	Princeton University	Feb 10 - June 30
Weinberger, Hans	University of Minnesota	
Yi, Fahuai	Suzhou University	Mar 9 - June 8

SHORT TERM VISITORS IN RESIDENCE

Anderson, Daniel	Northwestern University	Mar 10 - Mar 15
Barbu, Viorel	University of Cincinnati	Mar 24 - Mar 29
Beale, J. Thomas	Duke University	Mar 10 - Mar 22
Brattkus, K.	Cal Tech	Mar 10 - Mar 15
Brown, Robert A.	MIT	Mar 10 - Mar 15
Chen, Michael M.	Univ. of Illinois - Urbana	Mar 10 - Mar 15
Chiareli, Alessandra	Northwestern University	Mar 10 - Mar 15
Coriell, Sam	NIST	Mar 10 - Mar 15
Dandy, David	Sandia Nat. Labs.	Mar 10 - Mar 15
Davis, Steve	Northwestern University	Mar 10 - Mar 15
Derby, Jeffrey	University of Minnesota	Mar 10 - Mar 15
Dussan, Elizabeth B.	Schlumberger Doll Res. Center	Mar 10 - Mar 15
Gutierrez-Miravete, E.	Hartford Graduate Center	Mar 10 - Mar 16
Hassager, Ole	Technical University of Denmark	Mar 10 - Mar 15
Higdon, J.L.	University of Illinois, Urbana	Mar 10 - Mar 15
Higgins, Brian H.	Univ. of California-Berkeley	Mar 10 - Mar 15
Ho, Lee-Wing	Nektonics, Cambridge MA	Mar 9 - Mar 15
Hornung, Ulrich	U. der Bundeswehr München	Mar 10 - Mar 15
Howison, Sam	University of Oxford	Mar 10 - Mar 15
Hu, Bei	University of Notre Dame	Mar 10 - Mar 15
Huntley, Douglas	Northwestern University	Mar 10 - Mar 15
Johnson, Robert	U. of Illinois - Urbana	Mar 10 - Mar 16
Joseph, Daniel	University of Minnesota	Mar 10 - Mar 15
Kim, Sang-Tae	Univ. of Wisconsin-Madison	Mar 10 - Mar 15
Kistler, Stephan F.	3M	Mar 10 - Mar 15
Man, Chi-Sing	University of Kentucky	Mar 9 - Mar 13
Matarasso, Silvano	U. di Bologna	Mar 10 - Mar 15
McFadden, Geoffrey B.	NIST	Mar 10 - Mar 15
Miksis, Michael J.	Northwestern University	Mar 10 - Mar 15
Muller, U.	University of Karlsruhe	Mar 10 - Mar 15
Nochetto, Ricardo	University of Maryland	March 10-15, April 14-19
Ockendon, J. R.	University of Oxford	Mar 24 - Mar 29
Pozrikidis, C.	Univ. of Calif.-San Diego	Mar 10 - Mar 15
Reinelt, Douglas A.	Stanford University	Mar 9 - Mar 15
Rodrigues, José F.	Univ. Lisbon & CMAF	March 3 - March 16
Scott, Ridgway	University of Houston	Mar 10 - Mar 15
Scriven, L.E.	University of Minnesota	Mar 10 - Mar 15
Smith, Marc	Johns Hopkins University	Mar 9 - Mar 15
Solonnikov, V.A.	LOMI, USSR	Mar 11 - Mar 15
Tanveer, Saleh	Ohio State University	Mar 10 - Mar 15
Tavener, Simon	Penn State University	Mar 10 - Mar 15
True, Hans	Technical University of Denmark	Mar 10 - Mar 16
VandenBroeck, Jean-Marc	University of Wisconsin	Mar 10 - Mar 15
Wheeler, Adam A.	University of Bristol	Mar 10 - Mar 15
Worster, M. Grae	Northwestern University	Mar 10 - Mar 15
Young, Gerald	University of Akron	Mar 10 - Mar 15

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

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IMA NEWSLETTER #178

May 12 - May 29, 1991

1990-91 Program
PHASE TRANSITIONS AND FREE BOUNDARIES

NEWS AND NOTES

IMA Workshop: DEGENERATE DIFFUSIONS

May 13 - 18, 1991

Organizers: W.-M. Ni, L.A. Peletier, J.-L. Vazquez

The emphasis in this workshop will be on current and new problems in nonlinear diffusion equations involving free boundaries or sharp interfaces. Such problems include (i) higher order degenerate diffusion equations, such as the porous media equation, but of order 4, 6 or higher, (ii) backward/forward diffusion equations involving a regularization higher order term, (iii) strongly degenerate diffusion equations, of second as well as of higher order and (or) systems of degenerate equations.

The motivation for studying these problems is both mathematical and practical. Equations (i) arise in models for semiconductor fabrication and for behavior of viscous drops, a typical example of (ii) is the Cahn-Hilliard equation arising in the study of phase transitions. Equations of type (iii) arise in astronomy and also in phase transitions and degenerate systems arise in models in population dynamics, and in population biology.

In addition, some effort will be made to survey the state of the art of the more classical subjects on the theory of degenerate diffusion equations.

The study of degenerate diffusion equations is actively pursued in many places. The objective of this workshop is to provide some focus in this endeavor, and by inviting scientists and engineers as well as mathematicians, to keep it firmly linked to concrete problems.

The last two days of the workshop will be a celebration of Jim Serrin's sixty fifth birthday.

PARTICIPATING INSTITUTIONS:Georgia Institute of Technology, Indiana University, Iowa State University, Kent State University, Michigan State University, Northern Illinois University, Northwestern University, Ohio State University, Pennsylvania State University, Purdue University, University of Chicago, University of Cincinnati, University of Houston, University of Illinois (Chicago), University of Illinois (Urbana), University of Iowa, University of Kentucky, University of Manitoba, University of Maryland, University of Michigan, University of Minnesota, University of Notre Dame, University of Pittsburgh, Wayne State University
PARTICIPATING CORPORATIONS:Bellcore, Cray Research, Eastman Kodak, General Motors, Hitachi, Honeywell, IBM, Kao, Motorola, 3M, UNISYS

Minisymposium:
NONLINEAR DIFFUSION EQUATIONS & THEIR EQUILIBRIUM STATES

May 24 - 25, 1991

Cosponsored by the National Science Foundation & the IMA

Organizers: W.-M. Ni, L.A. Peletier, J. Serrin

Many problems in science and engineering require the analysis of mathematical models involving nonlinear diffusion equations, for example, equations of the form

$$u_t = \Delta u + f(u).$$

These can be studied either in bounded or unbounded domains under various different conditions on the boundary. Mathematically this leads to questions about existence, uniqueness and qualitative properties of the solutions. Depending on the diffusion operator or the source function f, free boundaries may occur.

Recently, research has focused on questions such as global existence (blow-up/extinction) and the rôle of self-similar solutions in the description of short- and large time behaviour of solutions, as well as detailed studies of the rich structure of the associated equilibrium problems. This involves ground states as well as nonlinear global bifurcation problems, in which the growth of the source term plays a critical rôle.

The present meeting is meant to offer an opportunity for the participants to learn of recent results and to interact with scientists and engineers

This is the third in a series of meetings on nonlinear partial- and ordinary differential equations related to diffusion equations. The first one of these was held in 1986, in Berkeley, and the second one in 1989, in Gregynog (Wales).

SCHEDULE FOR MAY 12 - MAY 29

IMA Workshop:
DEGENERATE DIFFUSIONS

May 13 - 18, 1991

Organizers: W.-M. Ni, L.A. Peletier, J.-L. Vazquez

Most of the program talks will be held in Conference Hall 3-180 on the entry floor of the Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

Monday, May 13

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:00 am	**Registration and coffee**	Reception Room EE/CS 3-176

9:30 am **Welcome and Orientation** Conference Hall EE/CS 3-180

9:40 am **Charles M. Elliott** Parabolic obstacle problems and phase transformations
University of Sussex

Abstract: We shall be concerned with the mathematical and numerical analysis of models of phase transformations based on the Ginzburg-Landau energy functional

1 $$E(u) = \int_\Omega \left[\frac{\gamma}{2}|\nabla u|^2 + \psi(u)\right] dx$$

where $\gamma > 0, \Omega \subset \mathbf{R}^m$ and $\psi(\cdot) : \mathbf{R}^m \to R$. The homogeneous energy $\psi(\cdot)$ is non-convex with at least two wells. The dynamic problem is given by the relaxation.

2 $$\frac{du}{dt} + DE(u) = 0$$

where D is an appropriate functional derivative. The scalar Cahn-Allen and Cahn-Hilliard equations

3 $$a)\ u_t - \gamma\Delta u + \psi'(u) = 0, \qquad b)\ u_t - \Delta(-\gamma\Delta u + \psi'(u)) = 0$$

are typical examples. One form for $\psi(\cdot)$ arising in the modelling of phase separation in a binary mixture is

4 $$\psi(u) = \frac{kT_c}{2} - kT\log_e 2 - \frac{1}{2}kT_c u^2 + \frac{1}{2}kT[(1-u)\log_e(1-u) + (1+u)\log_e(1+u)]$$

where k in Boltzmann's constant, T is absolute temperature and T_c is the critical temperature. Here $|u| \leq 1$ and the values $u = \pm 1$ correspond to the pure states of each component of the mixture. In the deep quench limit of $T/T_c \to 0$, it is appropriate to replace $\psi(\cdot)$ by

$$\psi(u) = \frac{1}{2} = \frac{1}{2}kT_c(1-u^2) + I_{[-1,1]}(u)$$

where $I_{[-1,1]}(u)$ is the indicator function of the interval $[-1,1]$.

In this talk we will show that the degenerate diffusion equation (3b) - (4) has a solution which converges in the deep quench limit to the solution of a parabolic double obstacle problem. We shall also consider the Cahn-Hilliard model for phase separation in a mixture with M components. Aspects of similar versions of the Cahn-Allen equation (3a) and the phase field equations will be discussed. Numerical simulations of the motion of phase boundaries using these models will be presented. Various parts of this work were performed in collaboration with J.F. Blowey, S. Luckhaus, Xinfu Chen and M. Copetti.

10:40 am **Coffee Break** Reception Room EE/CS 3-176

11:00 am **Paul Fife** Qualitative properties of Cahn-Hilliard dynamics
University of Utah

Abstract: Some important dynamical issues surrounding the Cahn-Hilliard equation will be delineated, and recent results addressing those issues will be presented. The dynamics of this equation is characterized by multiple time scales. Emphasis will be on short-time and intermediate-time, rather than long-time, phenomena.

Numerical Analysis Seminar

1:25 pm **Tsutomu Ikeda** Numerical approach to interfacial dynamics
Ryukoku University

The SEMINAR meets in Vincent Hall 570

2:00 pm	**Michiel Bertsch** U. di Roma II	A parabolic equation with a nonlinear dependence on the gradient

Abstract: We consider a one-dimensional nonlinear diffusion equation with vanishing diffusion coefficient for infinite gradients. In this lecture we shall explain the occurrence of discontinuous solutions, and we shall study the behaviour of solutions near their discontinuities. In addition we shall point out some delicate questions related to modelling aspects and well-posedness.

$$\text{SEMINAR IN } \begin{cases} \text{Mathematics} \\ \text{Physics} \\ \text{Ford Hall 150} \end{cases}$$

2:30 pm	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. I

Abstract: Quasi-exactly solvable problems are a new kind of spectral problems occupying an intermediate place between exactly solvable and non-solvable. The main property of them is the knowledge of a finite number of eigenstates. The simplest example is the potential

$$V = a^2x^6 + 2abx^4 + [b^2 - (2k+3)a]x^2$$

in which the first $N = [k/2] + 1$ eigenstates can be found algebraically. All one dimensional quasi-exactly solvable problems and all known exactly solvable ones have the same hidden symmetry group $SL(2, R)$. Quasi-exactly solvability is related to a new realization of hidden symmetry based on equivalence between the Schroedinger equation and a spectral problem for quadratic elements of the universal enveloping algebra in a representation in terms of differential operators of the first order. Using compact algebras leads to exactly solvable problems without separability of variables living on non-trivial manifolds.

This phenomenon has many connections with different branches of mathematics: spectral theory (a certain hint on classification of spectral problems), group representations, differential geometry, Riemannian geometry, theory of orthogonal polynomials (especially multi-dimensional polynomials). The most interesting results stem from special orbits of non-compact groups, like the conic of $SO(2,1)$, for which, in fact, there is no mathematical theory.

4:00 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

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Tuesday, May 14

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Sigurd Angenent** University of Wisconsin/IMA	Anisotropic motion by curvature
10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Robert V. Kohn** Courant Institute	Some open problems involving phase boundary motion by curvature related laws

Abstract: Two fundamentally different approaches are evolving for the mathematical description of phase boundary motion. One, based on viscosity solutions of Hamilton-Jacobi equations, has led to successful computations and mathematical theorems of existence and uniqueness. The other, based on singular limits of reaction diffusion equations, has had less spectacular success but seems in certain ways more flexible and closer to the physics. I will summarize these two approaches, then discuss a variety of open questions. Issues to be addressed include the following: A) Are there initial data for which the HJ and RD models of motion by mean curvature give different motion laws? B) Is the "slow motion manifold" program of Hale/Fusco

and Carr/Pego applicable to motion by mean curvature in several dimensions? C) What is the analogue of motion by mean curvature when there are three or more phases? D) How should one handle the motion of a curve or interface by mean curvature, when it is fixed at part of its boundary? E) In what sense should one interpret the "scaling law" that interfacial area decays as $t^{1/2}$ in a system evolving under motion by mean curvature?

Solid Mechanics Seminar

1:00 pm	**G.I. Barenblatt** USSR Academy of Science/IMA	Similarity and some relevant problems in fracture

The SEMINAR is in Akerman Hall 227

2:20 pm	**Nina N. Uraltseva** Lenigrad/Emory University	Evolution of nonparametric hypersurfaces with speed depending on the mean curvature

Abstract: In this joint work with Vladimir Oliker we consider an evolution which starts as a flow of graphs propagating in space with normal speed proportional to the mean curvature of the current hypersurface. G. Huisken has shown that if the boundary of the domain over which these graphs are considered satisfies the condition of J. Serrin (that is, it is convex "in the mean") then the corresponding initial boundary value problem with Dirichlet boundary data and smooth initial data admits a solution which exists for all time and smooth. Here, we consider the case of arbitrary domains with smooth boundaries not necessarily satisfying the condition of Serrin. Numerical experiments suggest that even if the flow starts with smooth initial data and homogeneous Dirichlet boundary data, singularities may develop in finite time at the boundary of the domain. However, we show that after a sufficiently large time the solution becomes again smooth and assumes the boundary values in classical sense. We also give sufficient conditions that guarantee existence of a solution which is smooth for all time $t \geq 0$. In addition, we give sharp estimates of the rate at which solutions tend to zero as $t \to \infty$.

3:30 pm **Contributed Papers**

Participants are invited to present contributed papers. Twenty minute time slots have been set aside for these talks. It is hoped that the speakers will leave five minutes of the time for discussion. Please contact the organizers if you wish to give a contributed paper.

3:30 pm	**Xinfu Chen** University of Minnesota	Phase field equations in the singular limit of sharp interface problems
3:50 pm	**Joost Hulshof** University of Leiden/IMA	The dipole solution for the porous media equation in N dimensions
4:10 pm		Fifteen minute break
4:25 pm	**Doug Meade** Purdue University	On the qualitative analysis for an epidemic model with directed diffusion
4:45 pm	**Marek Fila** University of Bratislava/IMA	Boundedness of global solutions of a degenerate diffusion equation
5:05 pm	**Amy Novick-Cohen** Technion	Sivashinsky type equations: one-dimensional steady states

AHPCRC Colloquium

3:30 pm	**Irene Martinez Gamba** Purdue University/IMA	Boundary behavior of solutions to equations modeling semiconductor devices

Abstract: We are going to discuss asymptotic singularities for the Drift-Diffusion two-dimensional model of a typical metal-oxide semiconductor (MOS), socks and boundary layer formation for one-dimensional hydrodynamic models.

The COLLOQUIUM is held in the AHPCRC Seminar Room 170
1100 South Washington Ave., Minneapolis

Wednesday, May 15

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Miguel Herrero** U. Complutense de Madrid	Some results on blow-up for semilinear parabolic problems

Abstract: We shall consider the problem

$$u_t = u_{xx} + f(u), \quad \text{where } f(u) - u^p (p > 1) \text{ or } f(u) = 0^u \tag{1}$$
$$u(x,0) = u_0(x) \geq 0 \tag{2}$$

let $u(x,t)$ be a solution of (1), (2) which blows up at, say, $x - 0$ and $t = T < +\infty$. A description of the possible blow-up profiles will be given and conditions under which some of these profiles occur will be discussed.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Victor Galaktionov** Keldysh Inst., USSR/IMA	On a blow-up set and monotonicity in time for a degenerate quasilinear heat equation

Abstract: The Cauchy problem for a quasilinear parabolic equation of nonlinear heat conduction type with a source is considered. Sharp estimates of the structure of blow-up sets are proved. It is shown that properties of monotone in time behavior of solutions at a fixed spatial point depends on the relationship betwen the nonlinear heat operator and the source and the behavior of the initial function in some neighborhood of this point. In particular, it is shown that arbitrary solution is strictly monotone in time at any point which is far enough from the initial support.

These results are proved by the method of intersection comparison either with the explicit nonimvariant solution having the same blow-up time or with continuous set of stationary solutions.

2:00 pm	**G.I. Barenblatt** USSR Acad. of Science	Mathematical model of capillary inbibition
3:30 pm	**Juan Luis Vazquez** U. Autonoma, Madrid/IMA	The Barenblatt equation of elasto-plastic filtration

Abstract: In this work with S. Kamin and L. Peletier we consider the equation

$$u_t + \gamma |u_t| = \Delta u\,, \quad 0 < |\gamma| < 1 \qquad (E)$$

in $Q = \{(x,t) : x \in \mathbf{R}^N, t > 0\}$. The equation arises in the theory of an elastic fluid in an elasto-plastic porous medium, under the assumption that the porous medium is irreversibly deformable, and was studied by Barenblatt and his collaborators since the '50s. On the other hand, we may write the equation in the form $u_t = \max\{L_1(u), L_2(u)\}$ if $\gamma < 0$, $u_t = \min\{L_1(u), L_2(u)\}$ if $\gamma > 0$ where $L_1(u) = \Delta u/(1-\gamma)$ and $L_2(u) = \Delta u/(1+\gamma)$. In this way we see that it is a particular case of the so-called parabolic Bellman equations of dynamic programming.

We construct for every $\gamma \in (-1,1)$ a selfsimilar solution of the form $B(x,t) = t^{-\alpha/2} f(\eta)$, with $\eta = x/t^{1/2}$, exhibiting a singularity at the origin $(x = 0, t = 0)$. This solution is unique up to a multiplicative constant. We show that when $\gamma \neq 0$ the similarity exponent α is *anomalous*, i.e., different from the usual $\alpha = N$ of the heat equation. More specifically, we prove that α is a strictly increasing function of γ, with $\alpha(0) = N$. Thus, B is a fundamental solution only if $\gamma = 0$. It is *very singular* if $\gamma > 0$, in the sense that the mass of the initial singularity is infinite, while for $\gamma < 0$ it is a *mildly singular* solution (i.e., the mass of the initial singularity is 0).

Our main result consists in proving that these selfsimilar solutions represent the asymptotic profiles to which every solution of (E) tends for large times, under the assumption that their initial data are nonnegative, continuous and fast decay as $x \to \infty$.

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Thursday, May 16

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

9:00 am	**Walter Littman** University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates is meeting on a regular basis with Professor W. Littman in Vincent Hall 570 in conjunction with the Industrial Problems Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific areas.

9:30 am	**Rui-Tao Dong** MSRI	Eigenfunctions and nodal sets on Riemannian manifolds

Abstract: I will discuss a BMO estimate of an eigenfunction of the Beltrami-Laplace operator, the Hausdorff measure of the nodal set of the eigenfunction and the connection between them.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Chang-Shou Lin** Nat'l Taiwan University/IMA	Uniqueness of solutions minimizing $\int_\Omega \lvert\nabla u\rvert^2/(\int_\Omega u^{p+1})^{\frac{2}{p+1}}$ in $\mathbf{R}^2$
2:00 pm	**Emmanuele DiBenedetto** Northwestern University	Analyticity of solutions to the fast diffusion equation

SEMINAR IN $\left\{\begin{array}{l}\text{Mathematics}\\\text{Physics}\\\text{Vincent Hall 570}\end{array}\right.$

2:30 pm	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. II

School of Mathematics Colloquium

3:30 pm	**Hiroshi Matano** University of Tokyo	A diffusion equation associated with equimeasurable rearrangement

Abstract: The theory of equimeasurable rearrangement – or symmetrization – has many applications in the calculus of variations, particularly in studying the spatial structure of global minimizers of certain energy functionals. The conventional theory of rearrangement, however, is of little use when it comes to local minimizers rather than the global ones. To fill the gap, one needs to establish a theory that would give an appropriate homotopy between a function u and its rearrangement u^*. The study of the nature of such "homotopy" leads us to investigating a certain diffusion equation under multiple integral constraints. We shall discuss the long-time behavior of solutions and stability of equilibria.

The COLLOQUIUM is held in Vincent Hall 16

Tea will be served at 3:10 pm in Vincent Hall 120

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Friday, May 17

Celebration in Honor of Professor James Serrin's 65th Birthday
Sponsored by the School of Mathematics and the IMA

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**S. Antman** University of Maryland	Nonlinear nonlocal problems of fluid-solid interactions

Abstract: A fluid flowing past a deformable boundary influences the shape of the boundary and, conversely, the shape of the boundary influences the flow. It is shown how such problems for steady (potential) flows (including cavitational flows) past nonlinearly elastic shells can be accurately formulated and globally analyzed. The need to determine the shape of the boundary introduces technical complications into the study of the flow that do not arise for problems with rigid boundaries.

10:15 am	**Coffee Break**	Reception Room EE/CS 3-176
10:45 am	**J. Nohel** University of Wisconsin, Madison	Quadratic systems of ODE's occurring in shear flows of non-Newtonian fluids

Abstract: After a brief discussion of known results and open problems for quadratic systems of ODE's in the context of Hilbert's sixteenth problem, we discuss qualitative results for two problems: (1) pressure-driven flows that are approximated by a one-parameter family of a system of two quadratic ODE's; (2) piston-driven flow that can be approximated by a system of four coupled quadratic ODE's. From the point of view of qualitative theory of ODE's, the most difficult issue is to rule out existence of periodic and homoclinic orbits in both problems. Problem (1) is solved completely by phase plane analysis that can be used to explain interesting phenomena in pressure-driven shear flows under cyclic loading and unloading. Problem (2) arose as an approximation for a mathematical model for a recent experiment in which nearly periodic disturbances were observed in order to maintain a fixed volumetric flow rate. The experiment is viewed as a feedback control problem: Find the driving pressure gradient (i.e., the feedback) subject to the constraint that the volumetric flow rate is fixed. The approximating system of four quadratic ODE's is globally well-posed in time. We present numerical and some qualitative results. Since none of the standard two dimensional theory is applicable, the global analysis presents formidable open problems that are of independent interest.

1:30 pm	**N. Trudinger** Australian National University	On viscosity solutions of uniformly elliptic fully nonlinear elliptic equations

Abstract: We consider the uniqueness and continuous differentiability of weak solutions (in the viscosity sense) of fully nonlinear elliptic equations satisfying natural structure conditions. The resultant derivative estimates extend earlier estimates of ours for classical solutions. As a special case we see that Lipschitz solutions of quasilinear elliptic equations are classical if the coefficients are sufficiently smooth.

2:15 pm	**Coffee Break**	Reception Room EE/CS 3-176
2:30 pm	**K. McLeod** University of Wisconsin, Milwaukee	Long-time behaviour of solutions of quasi-linear parabolic equations

Abstract: We will discuss some ongoing work (joint with A. Milani) for equations of the form

$$u_t - \sum_{i,j} a_{ij}(\nabla u) D^2_{ij} u = f,$$

where the right-hand side is in the Sobolev space H^s, s large.

3:15 pm	**Coffee Break**	Reception Room EE/CS 3-176V
3:45 pm	**R. Redheffer** UCLA	Some diverse but related nonlinear problems

Abstract: A variety of topics pertaining to nonlinear partial differential equations of the 2nd order are seen to be related, in that they can be discussed by similar methods and many of them have some connection with the work of James Serrin. The talk is, for the most part, expository.

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Saturday, May 18

Celebration in Honor of Professor James Serrin's 65th Birthday
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Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**P. Rabinowitz** University of Wisconsin, Madison	Multibump solutions of a semilinear elliptic equation on $\mathbf{R}^n$

Abstract: For a family of semilinear elliptic equations of which

$$* \qquad -\Delta u + u = a(x)|u|^{s-1}u$$

is a special case, we will discuss the existence of k bump solutions for all $k > 1$. In $(*)$, $a(x)$ is positive and periodic in the components of x and $1 < s < \frac{n+2}{n-2}$ if $n > 2$.

10:15 am	**Coffee Break**	Reception Room EE/CS 3-176
10:45 am	**Giovanni P. Galdi** University Degli Studi di Ferrara	Exterior stationary solutions to the Navier-Stokes equations with zero velocity at infinity

Abstract: We analyze the asymptotic structure of solutions to the Navier-Stokes equations past a three-dimensional body, subject to zero velocity at infinity and such that

$$\int_\Omega \nabla v : \nabla v < \infty,$$

where v is the velocity field (D-solution). Specifically, we show that if the data are small compared to the viscosity, every D-solution presents at large distances the same behaviour of the Stokes fundamental tensor and, in particular, it behaves like $0(1/|x|)$. Furthermore, we show that ***unless the data satisfy certain compatibility conditions*** v does not belong to Lebesgue spaces L^q, for all $g \in (1, 3]$ and, as a byproduct, that the kinetic energy of the fluid is infinite. On the other hand, such conditions are proved to be also sufficient to ensure the existence of solutions in these L^q-spaces. One of the basic results is obtained by adapting to the case at hand a well-known technique introduced by J. Serrin in 1963 within the context of uniqueness of unsteady flows.

1:30 pm	**J. Smoller** University of Michigan	Non singular static solutions of Einstein-Yang/Mills equations

Abstract: The only static solution to the vacuum Einstein equations is the celebrated Schwarzschild solution which is singular at $r = 0$. Similarly the pure Yang/Mills equations on $\mathbf{R}^4$ have no static, regular solutions, and if one couples Einstein's equations to Maxwell's equations, the only static solution is the Reissner-Nordström metric which is again singular at the origin. Finally, for any gauge group G, in $(2+1)$-space-time dimensions, the Einstein-Yang/Mills (EYM) equations likewise admit no static regular solutions. We prove that the contrary holds in $(3+1)$-space-time dimensions. In fact, with $SU(2)$ gauge group we prove that the EYM equations admit non-singular static solutions whose metric is asymptomatically flat (i.e., Minkowskian). Thus for non-abelian gauge fields, the Yang/Mills repulsive force can balance gravitational attraction, and prevent the formation of singularities in space-time.

2:15 pm	**Coffee Break**	Reception Room EE/CS 3-176
2:30 pm	**P. Pucci** University Degli Studi Modena	Precise damping conditions for global asymptotic stability of second order variational systems

Abstract: We discuss global asymptotic stability of the rest state for Euler-Lagrange systems of ordinary differential equations of the form

$$[\nabla G(u')]' + \delta(r)\nabla G(u') + f(r,u) = 0.$$

The most important conditions are that G be strictly convex in $\mathbf{R}^N$, $\nabla G(0) = 0$, and $(f(r,u),u) > 0$ for $u \neq 0$ and r sufficiently large. We obtain necessary and sufficient conditions for global asymptotic stability under general assumptions given in terms of the damping coefficient $\delta(r) \geq 0$. Some of the results also extend to general variational systems

$$\{g(r)\mathcal{F}_p(r,u,u')\}' = g(r)\mathcal{F}_u(r,u,u')$$

and even to quasi-variational systems in which the damping coefficient is allowed to depend on u'. The stability theorems generalize and include a number of previous results due to *Levin & Nohel, Smith, Salvadori, Artstein & Infante* and *Ballieu & Peiffer*. The main tool is a variational identity which supplies a useful new family of Liapunov functions.

3:15 pm	**Coffee Break**	Vincent Hall 502
3:45 pm	**W. Ziemer** Indiana University	The Dirichlet problem for functions of least gradient

Abstract: Let $\Omega \subset R^n$ be a bounded Lipschitz domain and let $g{:}\,\partial\Omega \to R^1$ be a continuous function. We consider the problem

$$\inf\left\{\|\nabla u\|(\Omega) : u \in BV(\Omega) \cap C^0(\overline{\Omega}),\ u = g \text{ on } \partial\Omega\right\}.$$

Here, $BV(\Omega)$ is the space of functions whose partial derivatives are measures and $\|\nabla u\|(\Omega)$ denotes the total variation of the vector-valued measure ∇u evaluated on Ω. We show that a solution $u \in BV(\Omega) \cap C^0(\overline{\Omega})$ exists provided $\partial\Omega$ satisfies two conditions, namely, that $\partial\Omega$ has non-negative mean curvature (in a weak sense) and that $\partial\Omega$ is not locally area-minimizing. Furthermore, if either condition fails, we show that there exists boundary data g for which the problem has no solution.

The existence of a solution is established by actually constructing each of its superlevel sets in such a way that they reflect the appropriate boundary conditions and that they are area-minimizing. Each superlevel set is a solution to a variational problem which involves the minimization of the perimeter of sets.

It is also shown that the solution is unique. Solutions can have regularity in the interior no better than that at the boundary. However, it is shown that if $g \in C^{0,\alpha}$, then the solution is $C^{0,\alpha/2}$ in the interior. These results are optimal.

Finally, we consider the problem

$$\inf\left\{\|\nabla u\|(\Omega) : u \in BV(\Omega),\ u = g \text{ on } \partial\Omega\right\}.$$

Under the added assumption that Ω satisfies an exterior ball condition at each of its boundary points, it is shown that the unique minimizer of the first problem is also the unique minimizer of the second. In particular, the solution of the second problem is necessarily continuous in $\overline{\Omega}$.

Buffet Dinner in Honor of James Serrin's 65th Birthday

6:00 pm	Marquette Place Apartments, 1314 Marquette Avenue South, Minneapolis MN, 35th floor Party Room

Cocktails are at 6:00 pm, dinner at 7:00. Reservations required.

Monday, May 20

SEMINAR IN $\begin{cases} \text{Free Boundary Problems} \\ \text{Vincent Hall 570} \end{cases}$

11:15 am **Alberto Tesei** — Conservation laws with source
Università Di Roma "La Sapienza"

Abstract: We study qualitative properties of solutions of the Cauchy problem

$$\begin{cases} u_t + u^{m-1}u_x = \lambda u^p & \text{in } (0,\infty) \times \mathbf{R} \\ u = u_0 & \text{in } \{0\} \times \mathbf{R} \end{cases}$$

where $m > 1, p > 1, \lambda = \pm 1$.

The meeting today is joint with the PDE Seminar

Numerical Analysis Seminar

1:25 pm **Elena Zampieri** — Finite element preconditioning for spectral approximations and application to linear elasticity
University of Minnesota

The SEMINAR meets in Vincent Hall 570

SEMINAR IN $\begin{cases} \text{Free Boundary Problems} \\ \text{Vincent Hall 570} \end{cases}$

2:30 pm **G.I. Barenblatt** — Evolution of the turbulent burst
USSR Acad. of Sciences/IMA

Real Analysis Seminar

3:00 pm **Luis Caffarelli** — Monge-Ampere equations and the mapping problem
Institute for Advanced Study

The SEMINAR meets in Vincent Hall 311

Tuesday, May 21

SEMINAR IN $\begin{cases} \text{Mathematics} \\ \text{Physics} \\ \text{Vincent Hall 570} \end{cases}$

9:30 am **A.V. Turbiner** — Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. III
ITEP, Moscow/CERN, Geneva

SEMINAR IN $\begin{cases} \text{Free Boundary Problems} \\ \text{Vincent Hall 570} \end{cases}$

11:15 am **Robert Kersner** — The disappearance of free boundaries
Hungarian Academy of Sciences/IMA

Abstract: This talk concerns joint works with P. DeMottoni and Sh. Kamin.

We investigate two models in which free boundaries disappear in finite time. The first model is the equation

$$u_t = \Delta u^m + f(x,t,u)$$

where $m > 1$ is a constant and the function f resembles the reaction term in Fisher's equation. The second model is

$$\rho(x)u_t = \Delta u^m$$

with $\rho(x)$ a positive function which decays to zero as $|x|$ tends to infinity.

AEM Fluid Mechanics Special Seminars

12:20 pm	**G.I. Barenblatt** USSR Acad. of Sciences/IMA	Turbulence in Stratified Fluids. I

The SEMINAR is held in Conference Hall EE/CS 3-180

IMA Postdoc Seminar

2:30 pm	**David Dobson** IMA	The time-harmonic Maxwell equations in a doubly periodic structure

Abstract: Consider the diffraction of a beam of particles in $\mathbf{R}^3$ when the dielectric coefficient is a constant ϵ_1 above a surface S and a (different) constant ϵ_2 below S, and the magnetic permeability is constant μ throughout $\mathbf{R}^3$. S is assumed to be a doubly periodic surface, say $z = f(x,y)$ with $f(x+mL_1, y+nL_2) = f(x,y)$ for all integers m,n. The existence and uniqueness of a solution satisfying a "radiation condition" at infinity is reduced to a system of Fredholm equations. Thus, for all but a discrete set of ϵ's there exists a unique solution. Joint work with Avner Friedman.

The Seminar meets in Vincent Hall 570

Special Lecture

2:30 pm	**André Avez** Paris	Symplectic geometry and quantum mechanics

The Lecture meets in Vincent Hall 314

Special Math/Physics Lecture

3:35 pm	**Abraham A. Ungar** North Dakota State University	Group-like structure underlying the c-ball in real inner product spaces

Abstract: A group-like structure underlying the c-ball V_c,

$$V_c = \{\sigma \in V_\infty : \|\sigma\| < c\}, \quad c = \text{ const. } > 0,$$

in an abstract real inner product space $V_\infty = (V_\infty, +, \cdot)$ is presented. The binary operation $+$ in V_∞ is no longer closed in V_c. Employing ideas from special relativity theory we define in terms of $+$ and $\cdot$ a binary operation $\oplus$ in V_c such that the pair $(V_c, \oplus)$ forms a group-like object which shares remarkable analogies with the group $(V_\infty, +)$ to which it reduces when $c \to \infty$. Applications to abstract group theory are demonstrated.

The LECTURE meets in Vincent Hall 570

Real Analysis Seminar

4:00 pm	**Luis Caffarelli** Institute for Advanced Study	Monge-Ampere equations and the mapping problem

The SEMINAR meets in Vincent Hall 314

Wednesday, May 22

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Roberta Dal Passo** IAC, Rome	Entropy solutions of a nonlinear parabolic equation: Uniqueness

Abstract: As in the case of first order conservation laws, certain parabolic equations, with vanishing diffusion coefficient for infinite gradients, admit weak solutions which may develop shocks. This phenomenon causes the nonuniqueness of the solution in distributional sense. Therefore it is necessary to impose some additional condition to determine uniquely a solution. We shall present such a condition, which is related to Oleinik's entropy condition, and which turns out to be necessary and sufficient.

THEORETICAL PHYSICS INSTITUTE SEMINAR

12:20 pm	**Joseph Fehribach** U. Alabama, Huntsville/IMA	Mullins-Sekerka stability analysis for melting-freezing waves in helium

Abstract: Helium, unlike any other material, exhibits the phenomenon of oscillatory melting-freezing waves at its solid-melt interface. This presentation will review the derivation of a moving boundary problem for this physical system, and will discuss the Mullins-Sekerka instability for this problem.

The SEMINAR is held in 435 Physics

Real Analysis Seminar

3:35 pm	**Umberto Mosco** University of Rome/IMA	Energy decay for degenerate Dirichlet forms of subelliptic type

The SEMINAR meets in Vincent Hall 113

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Thursday, May 23

SEMINAR IN { Mathematics / Physics / Vincent Hall 570

10:00 am	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. IV

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

11:15 am	**Irene Martinez Gamba** Purdue University/IMA	Boundary layer formation for viscosity approximations in transonic flow

Abstract: The boundary layer formation for viscosity approximations in one-dimensional transonic flow models in a bounded interval with a source or a force-collision term is considered. The energy equation is replaced by a pressure-density relationship. The values of the density are imposed at the endpoints.

It can be shown that the viscosity approximation converges to a weak solution satisfying the jump and classical entropy condition.

The boundary layer has a condition that determines the possible range of discontinuities for the density.

These models are used in steady-state hydrodynamic modelling of semiconductor devices and also in steady state nozzle flow where the source term accounts by geometrical effects.

Annual IMA Picnic

3:00 pm		Picnic at Como Dome Shelter in Como Park, St. Paul

School of Mathematics Colloquium

3:30 pm	**Luis Caffarelli** Institute for Advanced Study	The propagation of perturbations through solutions of 2nd order PDE's